12-3 Probability

Main Ideas

- Use combinations and permutations to find probability.
- Create and use graphs of probability distributions.

New Vocabulary

probability
success
failure
random
random variable
probability distribution
uniform distribution
relative-frequency histogram

GET READY for the Lesson

The risk of getting struck by lightning in any given year is 1 in 750,000. The chances of surviving a lightning strike are 3 in 4. These risks and chances are a way of describing the probability of an event. The **probability** of an event is a ratio that measures the chances of the event occurring.

Probability and Odds Mathematicians often use tossing of coins and rolling of dice to illustrate probability. When you toss a coin, there are only two possible outcomes—heads or tails. A desired outcome is called a **success**. Any other outcome is called a **failure**.

KEY CONCEPT — *Probability of Success and Failure*

If an event can succeed in s ways and fail in f ways, then the probabilities of success, $P(S)$, and of failure, $P(F)$, are as follows.

$$P(S) = \frac{s}{s+f} \qquad P(F) = \frac{f}{s+f}$$

Reading Math

Notation When *P* is followed by an event in parentheses, *P* stands for *probability.* When there are two numbers in parentheses, *P* stands for *permutations.*

The probability of an event occurring is always between 0 and 1, inclusive. The closer the probability of an event is to 1, the more likely the event is to occur. The closer the probability of an event is to 0, the less likely the event is to occur. When all outcomes have an equally likely chance of occurring, we say that the outcomes occur at **random**.

EXAMPLE 1 Probability with Combinations

Monifa has a collection of 32 CDs—18 R&B and 14 rap. As she is leaving for a trip, she randomly chooses 6 CDs to take with her. What is the probability that she selects 3 R&B and 3 rap?

Step 1 Determine how many 6-CD selections meet the conditions.

$C(18, 3)$ Select 3 R&B CDs. Their order does not matter.

$C(14, 3)$ Select 3 rap CDs.

Step 2 Use the Fundamental Counting Principle to find s, the number of successes.

$$C(18, 3) \cdot C(14, 3) = \frac{18!}{15!3!} \cdot \frac{14!}{11!3!} \text{ or } 297{,}024$$

(continued on the next page)

Step 3 Find the total number, $s + f$, of possible 6-CD selections.

$C(32, 6) = \frac{32!}{26!6!}$ or 906,192 $\quad s + f = 906{,}192$

Step 4 Determine the probability.

$P(\text{3 R\&B CDs and 3 rap CDs}) = \frac{s}{s+f}$ Probability formula

$= \frac{297{,}024}{906{,}192}$ Substitute.

≈ 0.32777 Use a calculator.

The probability of selecting 3 R&B CDs and 3 rap CDs is about 0.32777 or 33%.

CHECK Your Progress

1. A board game is played with tiles with letters on one side. There are 56 tiles with consonants and 42 tiles with vowels. Each player must choose seven of the tiles at the beginning of the game. What is the probability that a player selects four consonants and three vowels?

EXAMPLE 2 Probability with Permutations

Ramon has five books on the floor, one for each of his classes: Algebra 2, chemistry, English, Spanish, and history. Ramon is going to put the books on a shelf. If he picks the books up at random and places them in a row on the same shelf, what is the probability that his English, Spanish, and Algebra 2 books will be the leftmost books on the shelf, but not necessarily in that order?

Step 1 Determine how many book arrangements meet the conditions.

$P(3, 3)$ Place the 3 leftmost books.

$P(2, 2)$ Place the other 2 books.

Step 2 Use the Fundamental Counting Principle to find the number of successes.

$P(3, 3) \cdot P(2, 2) = 3! \cdot 2!$ or 12

Step 3 Find the total number, $s + f$, of possible 5-book arrangements.

$P(5, 5) = 5!$ or 120 $\quad s + f = 120$

Step 4 Determine the probability.

P(English, Spanish, Algebra 2 followed by other books)

$= \frac{s}{s+f}$ Probability formula

$= \frac{12}{120}$ Substitute.

$= 0.1$ Use a calculator.

The probability of placing English, Spanish, and Algebra 2 before the other four books is 0.1 or 10%.

CHECK Your Progress

2. What is the probability that English will be the last book on the shelf?

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Probability Distributions Many experiments, such as rolling a die, have numerical outcomes. A **random variable** is a variable whose value is the numerical outcome of a random event. For example, when rolling a die we can let the random variable D represent the number showing on the die. Then D can equal 1, 2, 3, 4, 5, or 6. A **probability distribution** for a particular random variable is a function that maps the sample space to the probabilities of the outcomes in the sample space. The table below illustrates the probability distribution for rolling a die. *A distribution like this one where all of the probabilities are the same is called a* ***uniform distribution****.*

Reading Math

Random Variables
The notation $P(X = n)$ is used with random variables. $P(D = 4) = \frac{1}{6}$ is read *the probability that D equals 4 is one sixth.*

D = Roll	1	2	3	4	5	6
Probability	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$

$P(D = 4) = \frac{1}{6}$

To help visualize a probability distribution, you can use a table of probabilities or a graph, called a **relative-frequency histogram**.

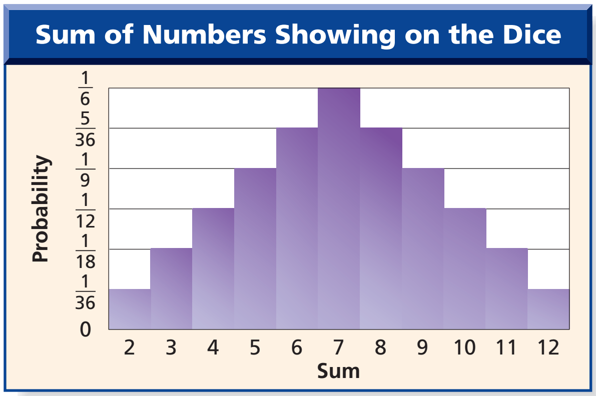
EXAMPLE Probability Distribution

3 **Suppose two dice are rolled. The table and the relative-frequency histogram show the distribution of the sum of the numbers rolled.**

Reading Math

Discrete Random Variables A discrete random variable is a variable that can have a countable number of values. The variable is said to be *random* if the sum of the probabilities is 1.

S = Sum	2	3	4	5	6	7	8	9	10	11	12
Probability	$\frac{1}{36}$	$\frac{1}{18}$	$\frac{1}{12}$	$\frac{1}{9}$	$\frac{5}{36}$	$\frac{1}{6}$	$\frac{5}{36}$	$\frac{1}{9}$	$\frac{1}{12}$	$\frac{1}{18}$	$\frac{1}{36}$



a. Use the graph to determine which outcome is most likely. What is its probability?

The most likely outcome is a sum of 7, and its probability is $\frac{1}{6}$.

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b. Use the table to find $P(S = 9)$. What other sum has the same probability?

According to the table, the probability of a sum of 9 is $\frac{1}{9}$. The other outcome with a probability of $\frac{1}{9}$ is 5.

CHECK Your Progress

3A. Which outcome(s) is least likely? What is its probability?

3B. Use the table to find $P(S = 3)$. What other sum has the same probability?

CHECK Your Understanding

Example 1 (pp. 697–698)

Suppose you select 2 letters at random from the word *compute*. Find each probability.

1. P(2 vowels) **2.** P(2 consonants) **3.** P(1 vowel, 1 consonant)

Example 2 (pp. 698–699)

ORGANIZATION **An administrative assistant has 4 blue file folders, 3 red folders, and 3 yellow folders on her desk. Each folder contains different information, so two folders of the same color should be viewed as being different. She puts the file folders randomly in a box to be taken to a meeting. Find each probability.**

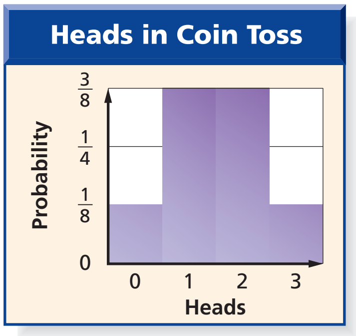
4. P(4 blue, 3 red, 3 yellow, in that order)

5. P(first 2 blue, last 2 blue)

Example 3 (pp. 699–700)

The table and the relative-frequency histogram show the distribution of the number of heads when 3 coins are tossed. Find each probability.

H = Heads	0	1	2	3
Probability	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$



6. $P(H = 0)$

7. $P(H = 2)$

Exercises

HOMEWORK HELP	
For Exercises	See Examples
8–15	1
16–21	2
22–27	3

Bob is moving and all of his sports cards are mixed up in a box. Twelve cards are baseball, eight are football, and five are basketball. If he reaches in the box and selects them at random, find each probability.

8. P(3 football) **9.** P(3 baseball)

10. P(1 basketball, 2 football) **11.** P(2 basketball, 1 baseball)

12. P(1 football, 2 baseball) **13.** P(1 basketball, 1 football, 1 baseball)

14. P(2 baseball, 2 basketball) **15.** P(2 football, 1 hockey)

DVDS **Janice has 8 DVD cases on a shelf, one for each season of her favorite TV show. Her brother accidentally knocks them off the shelf onto the floor. When her brother puts them back on the shelf, he does not pay attention to the season numbers and puts the cases back on the shelf randomly. Find each probability.**

16. P(season 5 in the correct position)

17. P(seasons 1 and 8 in the correct positions)

18. P(seasons 1 through 4 in the correct positions)

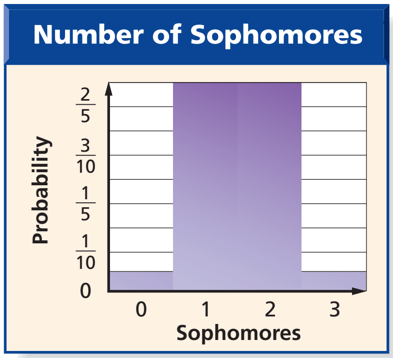
19. P(all even-numbered seasons followed by all odd-numbered seasons)

20. P(all even-numbered seasons in the correct position)

21. P(seasons 5 through 8 in any order followed by seasons 1 through 4 in any order)

Three students are selected at random from a group of 3 sophomores and 3 juniors. The table and relative-frequency histogram show the distribution of the number of sophomores chosen. Find each probability.

Sophomores	0	1	2	3
Probability	$\frac{1}{20}$	$\frac{9}{20}$	$\frac{9}{20}$	$\frac{1}{20}$



22. P(0 sophomores)

23. P(1 sophomore)

24. P(2 sophomores)

25. P(3 sophomores)

26. P(2 juniors)

27. P(1 junior)

Real-World Career

Physician

In addition to the MCAT, most medical schools require applicants to have had one year each of biology, physics, and English, and two years of chemistry in college.

Math Online

For more information, go to algebra2.com.

28. **LOTTERIES** The state of Texas has a lottery in which 5 numbers out of 37 are drawn at random. What is the probability of a given ticket matching all 5 numbers?

ENTRANCE TESTS **For Exercises 29–31, use the table that shows the college majors of the students who took the Medical College Admission Test (MCAT) recently.**

If a student taking the test were randomly selected, find each probability. Express as decimals rounded to the nearest thousandth.

Major	Students
biological sciences	15,819
humanities	963
math or statistics	179
physical sciences	2770
social sciences	2482
specialized health sciences	1431
other	1761

29. P(math or statistics)

30. P(biological sciences)

31. P(physical sciences)

32. **CARD GAMES** The game of euchre (YOO ker) is played using only the 9s, 10s, jacks, queens, kings, and aces from a standard deck of cards. Find the probability of being dealt a 5-card hand containing all four suits.

EXTRA PRACTICE

See pages 917, 937.

Math Online

Self-Check Quiz at algebra2.com

33. **WRITING** Josh types the five entries in the bibliography of his term paper in random order, forgetting that they should be in alphabetical order by author. What is the probability that he actually typed them in alphabetical order?

H.O.T. Problems

34. **OPEN ENDED** Describe an event that has a probability of 0 and an event that has a probability of 1.

CHALLENGE **Theoretical probability** is determined using mathematical methods and assumptions about the fairness of coins, dice, and so on. **Experimental probability** is determined by performing experiments and observing the outcomes.

Determine whether each probability is *theoretical* or *experimental*. Then find the probability.

35. Two dice are rolled. What is the probability that the sum will be 12?

36. A baseball player has 126 hits in 410 at-bats this season. What is the probability that he gets a hit in his next at-bat?

37. A hand of 2 cards is dealt from a standard deck of cards. What is the probability that both cards are clubs?

38. ***Writing in Math*** Use the information on page 697 to explain what probability tells you about life's risks. Include a description of the meaning of *success* and *failure* in the case of being struck by lightning and surviving.

STANDARDIZED TEST PRACTICE

39. ACT/SAT What is the value of $\frac{6!}{2!}$?

A 3

B 60

C 360

D 720

40. REVIEW A jar contains 4 red marbles, 3 green marbles, and 2 blue marbles. If a marble is drawn at random, what is the probability that it is *not* green?

F $\frac{2}{9}$ **G** $\frac{1}{3}$ **H** $\frac{4}{9}$ **J** $\frac{2}{3}$

Spiral Review

Determine whether each situation involves a *permutation* or a *combination*. Then find the number of possibilities. (Lesson 12-2)

41. arranging 5 different books on a shelf

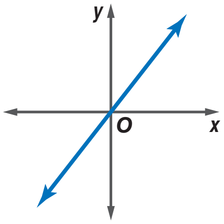
42. arranging the letters of the word *arrange*

43. picking 3 apples from the last 7 remaining at the grocery store

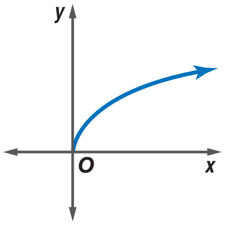
44. How many ways can 4 different gifts be placed into 4 different gift bags if each bag gets exactly 1 gift? (Lesson 12-1)

Identify the type of function represented by each graph. (Lesson 8-5)

45.



46.



GET READY for the Next Lesson

PREREQUISITE SKILL **Find each product if $a = \frac{3}{5}$, $b = \frac{2}{7}$, $c = \frac{3}{4}$, and $d = \frac{1}{3}$.**

47. ab **48.** bc **49.** cd **50.** bd **51.** ac